POLYHEDRAL PROJECTIONS.

John Wilkes 10 Feb 2010

I wish to bring this report on water and our beginnings with research to a close, by describing some aspects of very early studies in which I was involved under the tutelage of George Adams. My move to London in 1953 to attend the Royal College of Art Sculpture School brought me increasingly into contact with the work of Rudolf Steiner (this I had met through Dr Ernst Lehrs in 1951) through people who had known and studied under him.

I was in the process of writing my final thesis and came to George Adams for advice. My theme was 'The Living Nature of Form'. Among many other subjects, we were studying classical Geometry with Architect Sergei Kadleigh and when I mentioned this George Adams generously offered to instruct me in some aspects of Modern Synthetic (in contrast to Analytic) Projective Geometry. This geometry is not concerned with number but with process and is ideal for use within the study of nature.

We began with the study of polyhedra and methods of bringing them into movement through for instance the conic sections, sphere, ellipsoid, paraboloid and hyperboloid. For this purpose it is necessary to determine the origin of such three dimensional forms, within the two dimensional plane. All form archetypes are two dimensional.
For instance with respect to a cube (hexahedron) we find it is generated from three points at infinity (within the Harmonic Net). If we look at it edgewise we see three sets of four parallel lines, indicating the three directions of space.

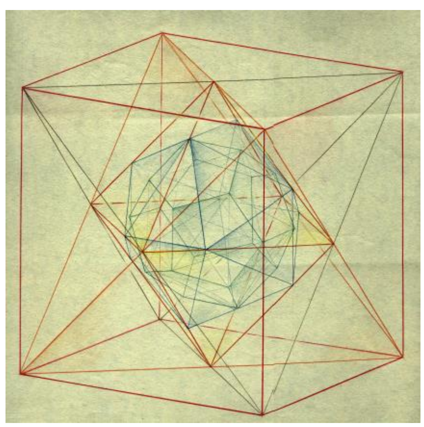
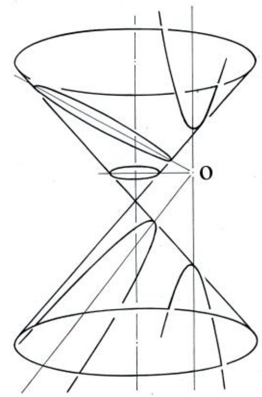


Fig.1: All Platonics (drawn) and a multitude of other polyhedra sitting within a cube.

Very soon we find that not only all the other Platonics sit within the cube but a multitude of increasingly complex polyhedra, they are potentially present even if not physically manifest (Fig.1). They all originate in the same plane at infinity creating an increasingly intensive 'field of form' as one approaches the infinity within; we could also describe this as the central point of the whole system. Everything physical is taking place between the periphery which expresses total expansion, and a central point which expresses total contraction.

If we now imagine the flat plane at infinity moving inwards, this will take place from the physical direction we chose, the spherical envelope of the cube will move out in the opposite direction, through the conic sections, while the infinite point within will remain fixed. First of all the sphere will become slightly ellipsoidal.   As the ellipsoid extends outwards with increasing speed its extremity will eventually reach infinity and become a paraboloid, beyond which it becomes an hyperboloid. This circumscribing envelope carries within it the polyhedral projections. A fascinating aspect of this process is that we have one spherical and one parabolic form, while ellipsoid and hyperboloid move through a vast range of forms.(Fig 2).

Described from a planar point of view we see that the circle only occurs at right angles to the cone axis, as this angle changes from ‘0’ the ellipse shoots out becoming longer until the angle is parallel to the cone, when the parabola appears. Beyond this the plane of the hyperbola cuts both extremities of the cone moving through its mid position when parallel to the cone axis.



(Fig 2) Sections of the cone.

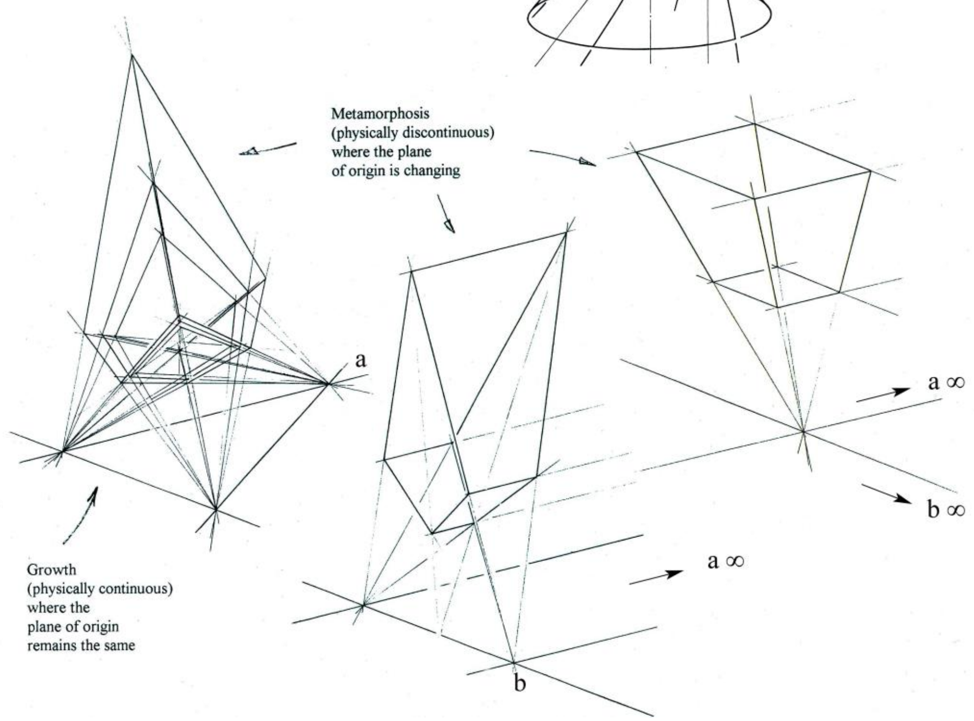


Fig.3 above: left: Projection process starts from three points in the plane of origin (called the Harmonic Net or 13 Configuration) and shows continuous growth while the three points remain fixed. Rotation of the projection anti-clockwise leads point ‘a’ towards infinity as in the **middle** drawing, (a ∞) rotation of this projection from front to back anti-clockwise will move point ‘b’ out to infinity, right drawing.(b ∞)

On the left in Fig.3 a projection process is shown from a fixed set of three points, which will grow through the conic sections. The beginning is similar to the opening of a bud. If the projection is rotated towards the left (anti-clockwise) while keeping the plane of origin in place, the point ‘a’ on the right will move out towards infinity. When this is reached the nature of the projection will be as the central drawing ‘a ∞’(Fig.3) Likewise if this projection is rotated from front to back (anti-clockwise) the nearest point ‘b’ at the front will move out to infinity ‘b ∞’ (Fig.3 right). When this is reached the third drawing will appear with one physical point of origin and two no longer physical but infinitely far away ‘a ∞’ and ‘b ∞’. If the plane in which this last point is resting is now moved out to infinity the projection can become a regular cube as in (Fig 1).

The left drawing in Fig.3 shows the possibility of growth while the origin remains fixed. By means of rotation the plane of origin changes with the projections taking on a metamorphic relationship. The reason for this is that a change takes place within the two dimensional origin, which represents the none-physical plane at infinity. When re-entering the three dimensional, the projection has changed.

If we consider one leaf, growth which is essentially continuous, brings about changes in the shape of the physical substance (Fig.4b lower). If we look at the next leaf, something much more dramatic has taken place in the none physical gap, which can bring about a new leaf which also grows but shows quite different characteristics. Growth can be described as a physically continuous process whereas metamorphosis is essentially a physically discontinuous process (Fig.4b upper).

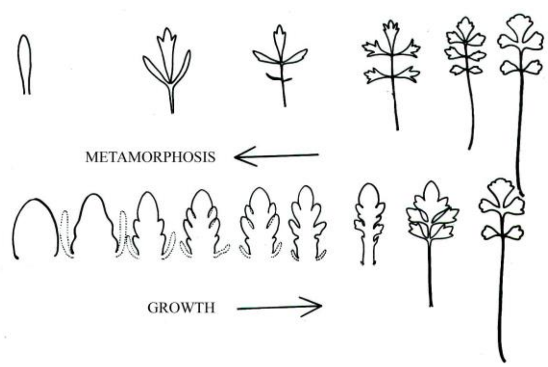


Fig.4: b (lower drawing) as example the continuous growth of the cress leaf finally brings about the adult leaf. One can imagine the archetype remains the same. Any number of drawings can be used to express this process in time. It is hidden from our consciousness, we are only aware of the final result.

(Above) Fig.4:b upper drawing illustrating the processes of metamorphosis where we imagine the archetype is changing, here we are experiencing a fixed number of stages, which are visible in space.

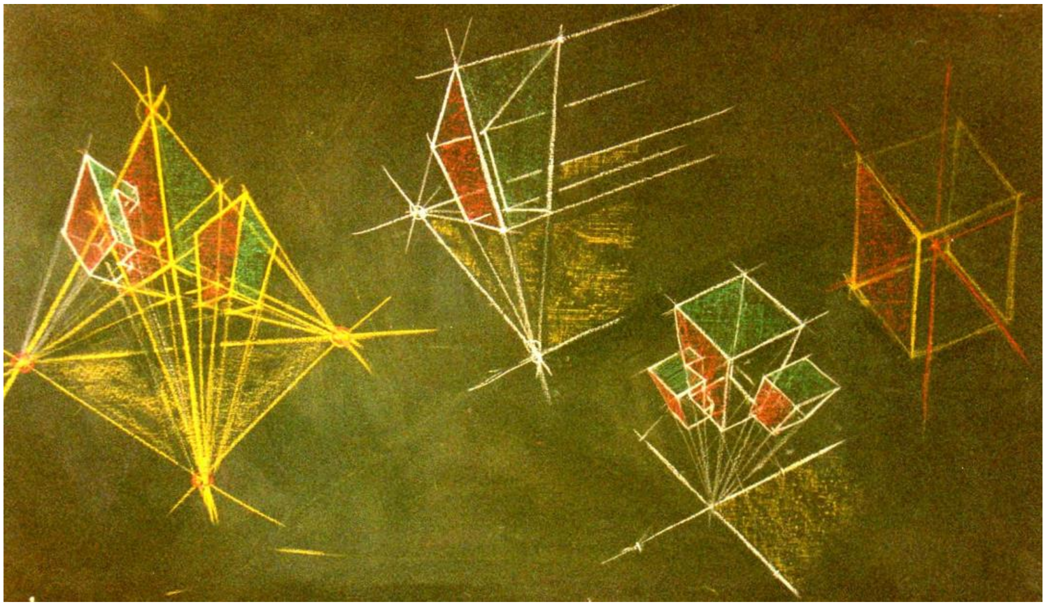
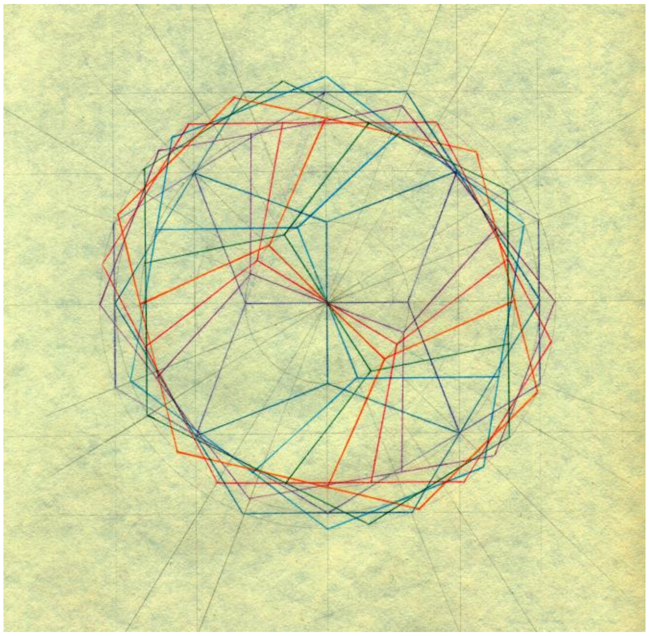


Fig.5: blackboard drawing illustrating the main symmetries with point (left), line (2nd from left) and plane (3rd from left), and the projection of a regular cube (right).

This freehand blackboard drawing (Fig.5) shows further possibilities with projections. Here the three main symmetries are illustrated with point, line and plane plus on the right the regular type cube projection from the three infinite points (this last drawing is not quite correct in the proportions but suffices to show the principle).

If we move on to the Dodecahedron (Platonic solid composed of twelve regular pentagonal faces), which is essentially five cubes (twelve pentagrams) we have a rotational process which is obviously more complex.

If we first consider a regular Dodecahedron section and rotate it (Fig. 6 left), it retains its shape through six positions because its origin we find in the plane at infinity, all relevant edges (pentagon and pentagram) are parallel to each other. If we now bring in that plane from infinity and express it as a line, it contains the points from which the ellipsoidal projection is built (Fig.6 right). Each dodecahedral projection has changed its shape because the points of origin have moved progressively along the line of origin at the bottom of Fig.6 (right).



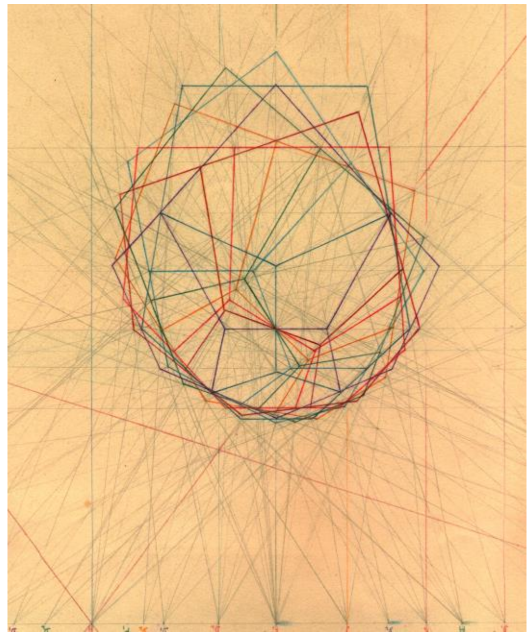
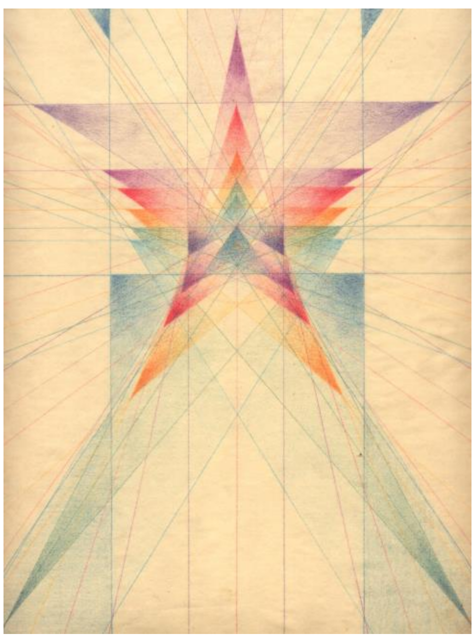


Fig.6: left rotation of a regular Dodecahedron section originating from a plane at infinity, **right** Dodecahedral projections change within the ellipse as the points of origin move along the line of origin, brought inwards and visible at the bottom of the drawing .

When this line of origin is considered as the archetypal plane (edgewise), it appears in plan as a metamorphic sequence of pentagrams, the points of which I discovered, move on hyperbolae (Fig.7). Having been instructed by George Adams how to draw the individual projections, I could not wait to bring them all into one drawing in order to experience how they related, something that had apparently not yet been attempted. The result was for me astonishing. First of all I produced a quarter rotation of the Dodecahedron (Fig. 7 left), then a half rotation of the Dodecahedron (Fig.7 right). These drawings were produced for the first time in 1956 (Fig7).



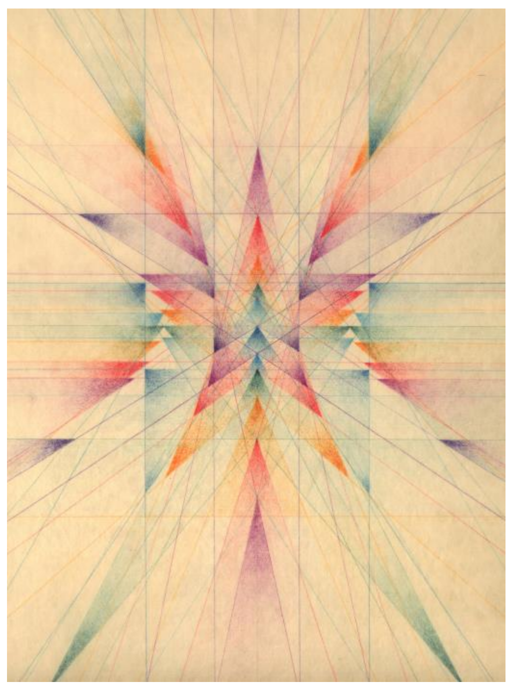


Fig. 7: left 90° rotation **right** 180°

Drawn individually in perspective it is possible to produce two dimensional representations of three dimensional projections of the polyhedra see figures 9, 10 and 11 below. Fig.8 Below is shown an example of the Icosahedron and within this its dual Dodecahedron (blue) and finally a further Icosahedron (red) (Fig.8). This process continues towards the point at infinity within and also to the plane at infinity outwards.

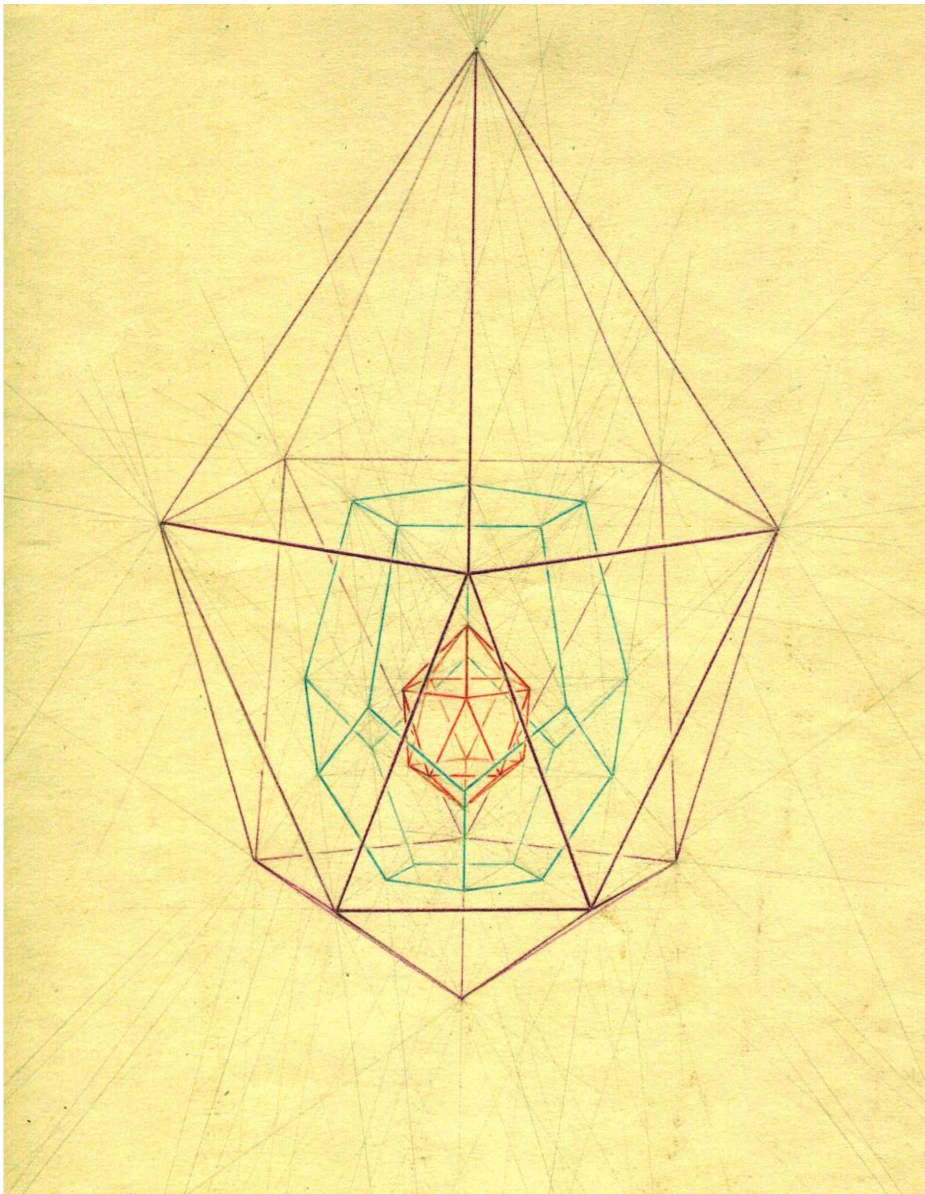


Fig.8: Outer Icosahedron with inner dual Dodecahedron (blue) and finally the further Icosahedron (red). These metamorphic projections continue towards the point at infinity inwards and to the plane at infinity outwards.

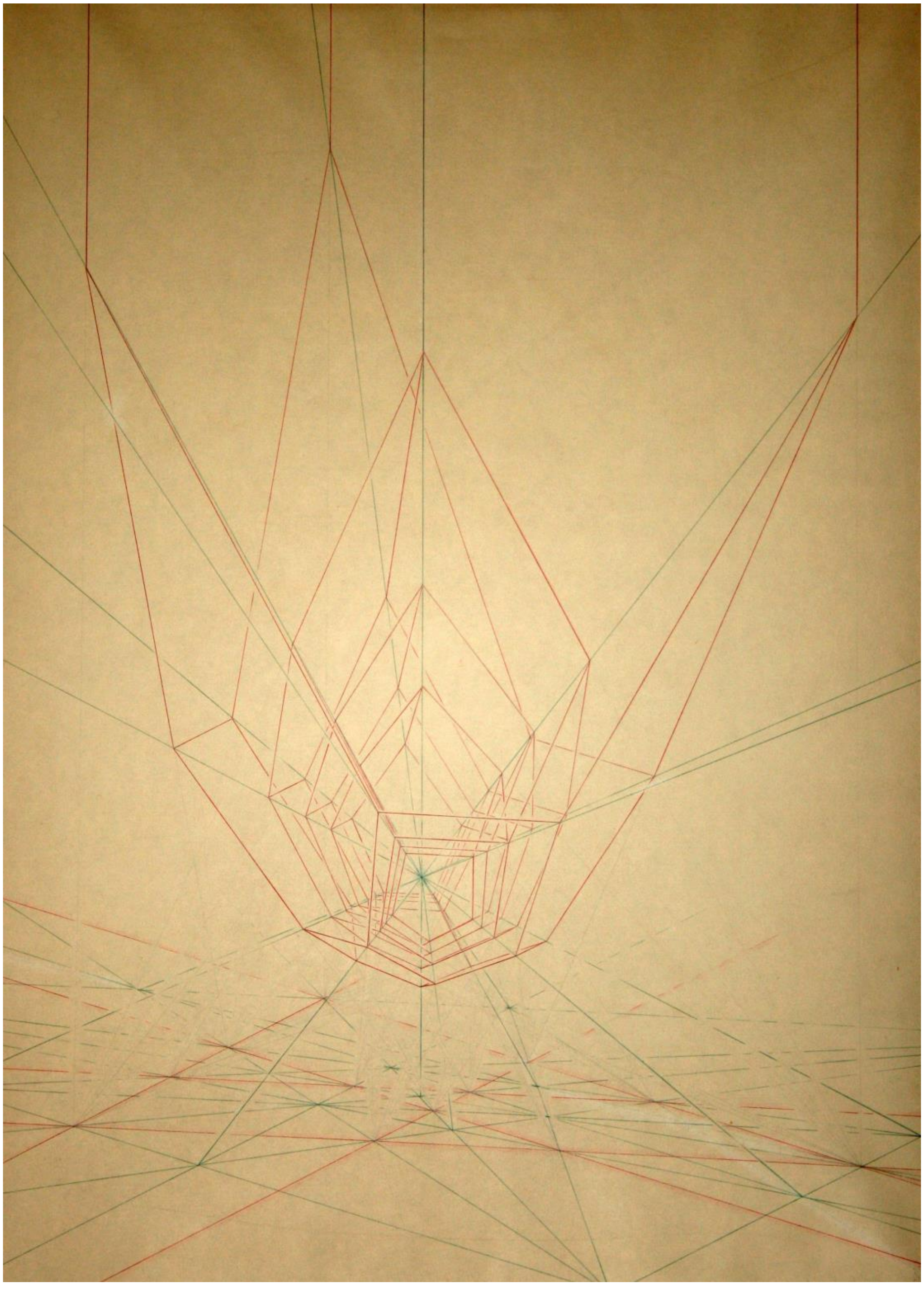


Fig.9: Growing projections from a fixed archetypal plane, about the apex point showing threefold symmetry due to the triangular orientation of the archetypal plane. This is visible as a red triangle below the projections.

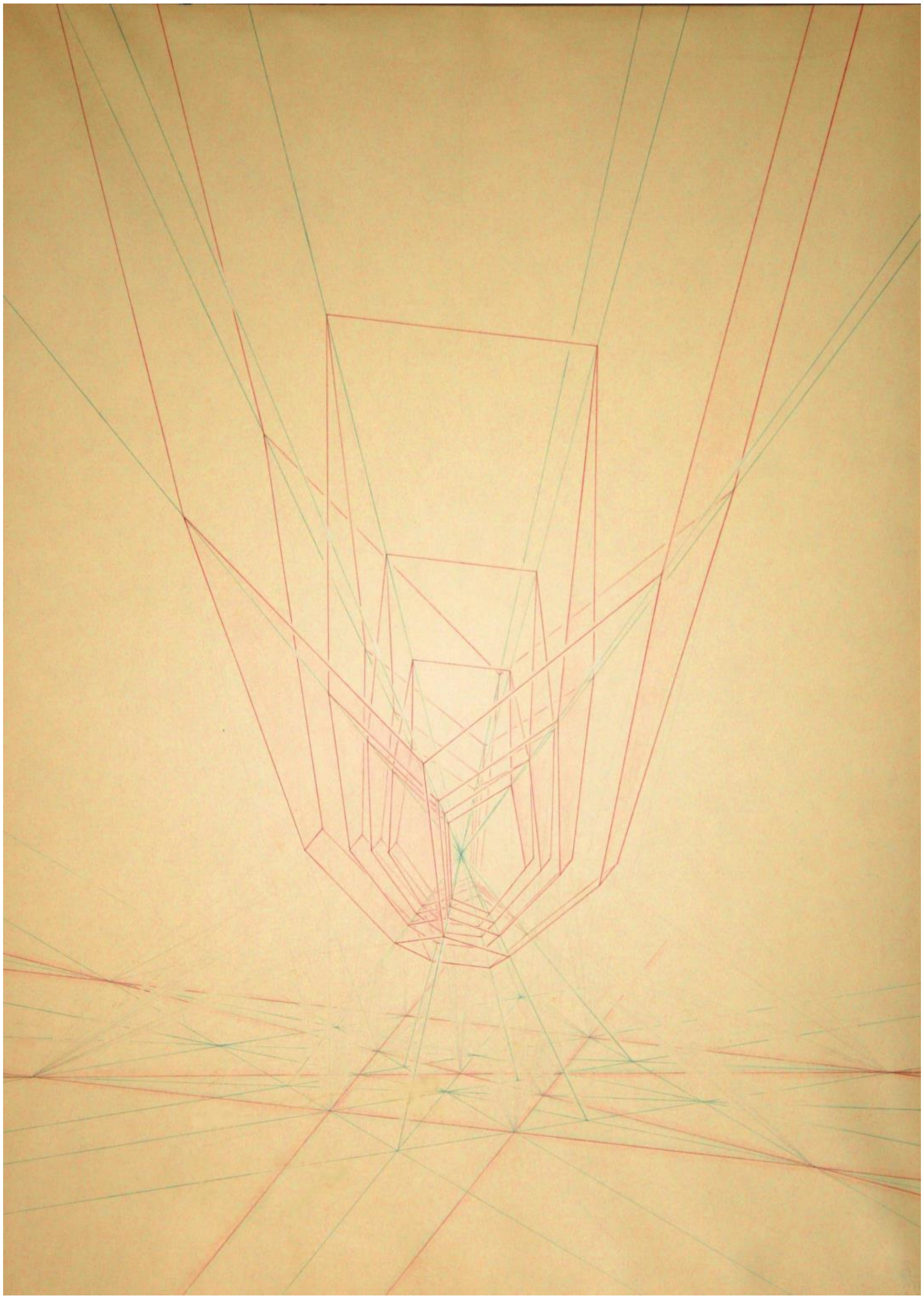
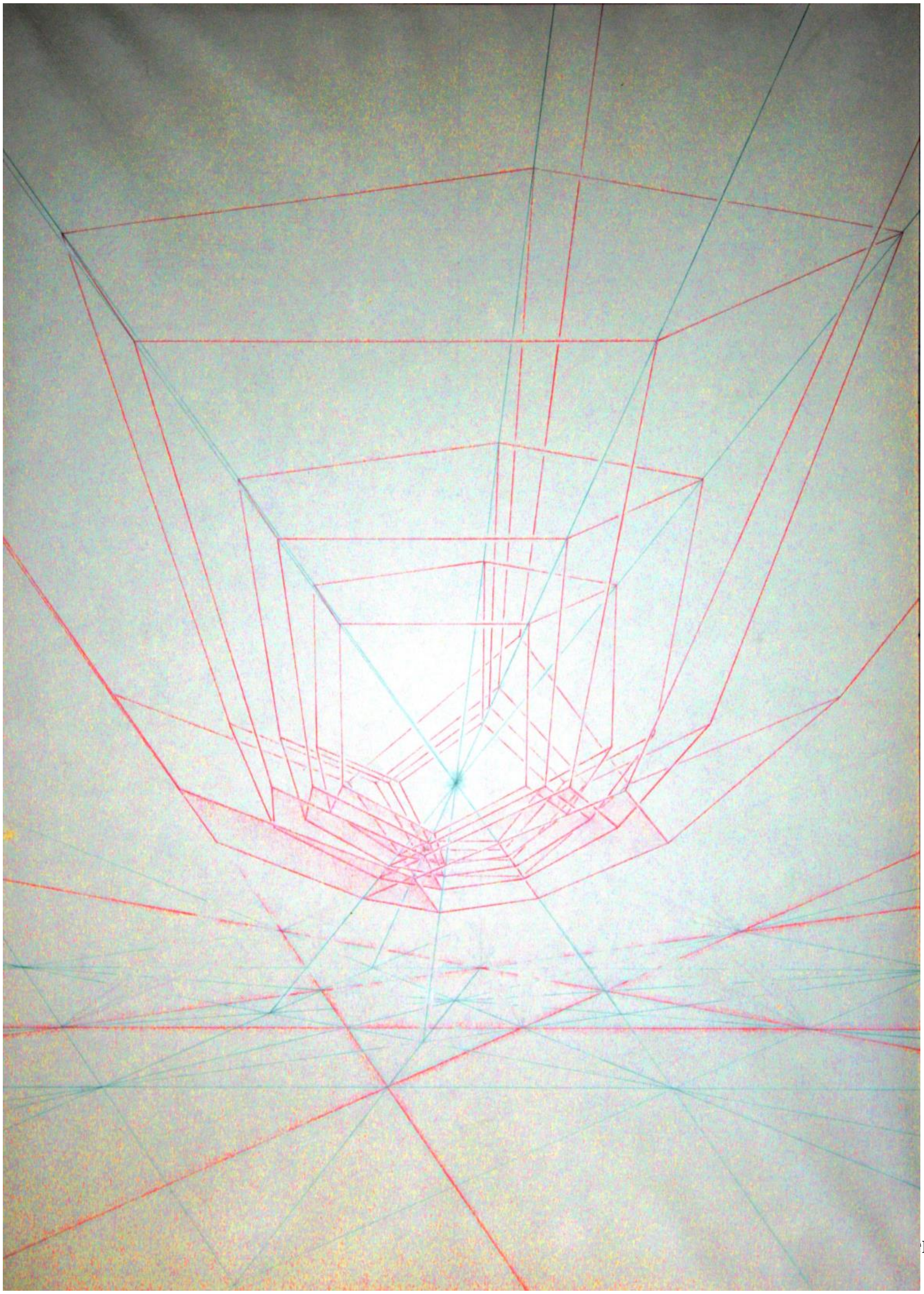


Fig.10 Growth projections relative to the top edge showing twofold symmetry. After rotation of the system in Fig.9, the plane of origin reaches a position where a rectangle is visible. This is visible as sets of parallel red lines in the archetypal plane from which the projections emanate.



relative to point (Fig.9), line (Fig.10) and plane (Fig.11). Respectively they show a threefold symmetry, a twofold symmetry (here the two symmetries are different and perpendicular to each other) and a fivefold symmetry.

These projections are intended to show a situation when the central vertical axis is perpendicular to the plane of origin. This central axis however can be chosen inclined to the plane of origin, then the three dimensional projections will tend naturally to be asymmetrical while maintaining their special characteristics. The whole of space can be filled with such projections from each position of the archetype (plane of origin). Within each drawing, a similar process to that of the growth of a bud is demonstrated. However, once more to be clear, the three separate drawings (Fig's 9, 10 and 11) demonstrate a 'rotational metamorphosis'.
The plane of origin in the last drawing is clearly a pentagram (red in Fig.11). When the system is rotated the plane of origin reaches a position where a rectangle is visible (red in Fig.10). On further rotation a triangular position is reached (red in Fig.9), from these three points for instance the apex of the projection is chosen.
It is clearly visible in the three separate metamorphosing projections, that the three stages of growth in each are so designed that the fourth position reaches infinity. This is called a growth measure and can be designed as desired before starting. As a process it brings a specific order into the drawings.

There is very much more to bring to this subject but this would take us far beyond the scope of this article.